Hierarchical Search on DisCSPs

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Introduction

- What is Constraint Satisfaction?
- What is Distributed CSP?
- Why do we use search?
- Why binary CSPs?

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Constraint Satisfaction Problems

Sudoku as a CSP

- n^4 variables in $n^2 \times n^2$ grid
- Domain of each variable: $\{1, \ldots, n^2\}$, except for the "open" cells
- Constraints: values for each row, column, major $n \times n$ block are *alldifferent*

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Sudoku as a CSP

			7	5				
	3			4	8		2	
1								6
	4							8
7	9						3	1
2							7	
5								7
	8		3	2			4	
				6	9			

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Distributed CSPs

- ABT is the classical approach [Yokoo and Hirayama, 2000]
- Timetabling
- Privacy considerations
- Message delays
- Algorithm performance: CCs, messages [Meisels et al., 2002]

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Motivation

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Motivation

- More parallelism and less backtracking
- Hierarchy facilitates smart PAs combination
- Independency of hierarchy and search

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Motivation: Combining PAs

- Theoretic motivation of DisHS
- *Constraint weight:* probability that a pair of values is not in conflict
- *Virtual constraint:* combination of constraints, weight approximated by multiplication
- Expected CCs when ordering k constraints by weight:

$$1 + w(k-1) \leq E_k(w) \leq \frac{1-w}{1-\sqrt[k]{w}}$$

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Algorithms Overview

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Hierarchical Search

- Partition
- Distributed Hierarchical Search
- Descending Requirements Search

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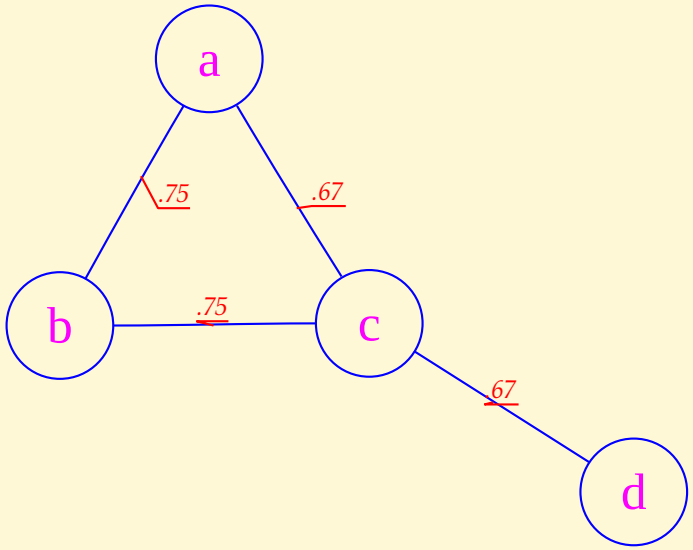
Example: Partition

- Consider a small graph-coloring problem
- Domains of a, c and d are $\{1, 2, 3\}$
- Domain of b is $\{1, 2, 3, 4\}$



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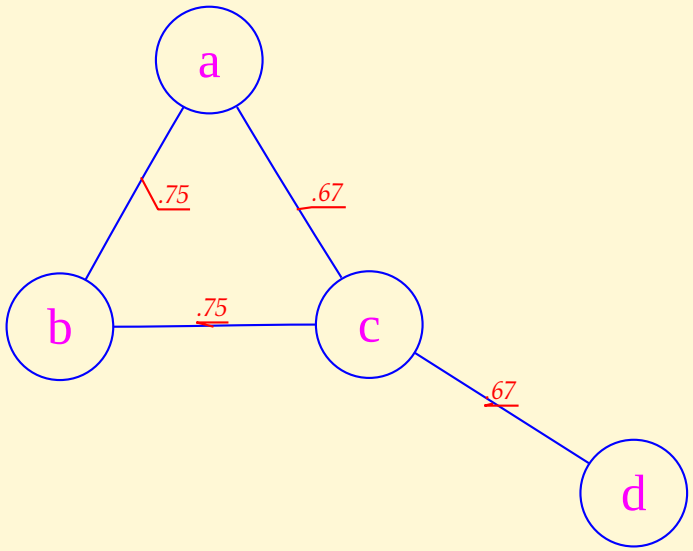
Example: Partition

- Each agent sends Join to a minimal-weight neighbor
- $a \rightarrow c$, $b \rightarrow a$, $c \rightarrow d$, $d \rightarrow c$
- c and d join, send each other components info
- c sends Done to a, b



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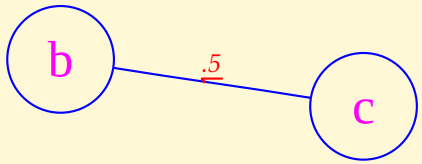
Example: Partition

- *a* and *b* remove *c* from neighbors list
- *a* and *b* join, and send Done to *c*
- *a*, *b*, *c*, *d* send Leader messages to chosen leaders
- Leaders are activated at the next level (can't be confused by previous levels messages)



- *b* and *c* join, picking leader *d*

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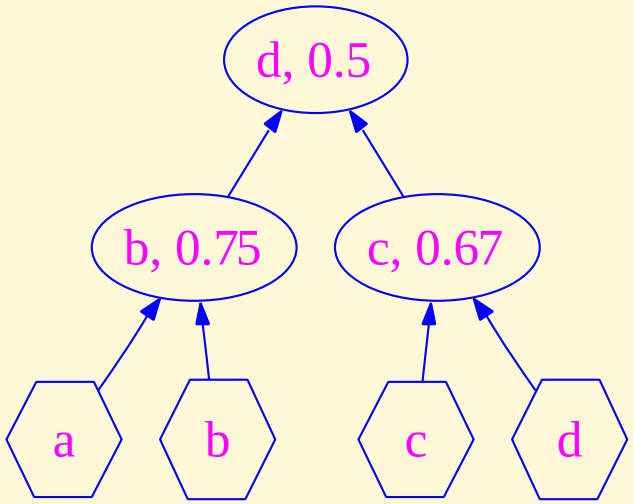
Example: Partition

- We now have a hierarchy of agents
- *d* sends Search message to all agents in order to initiate the search process



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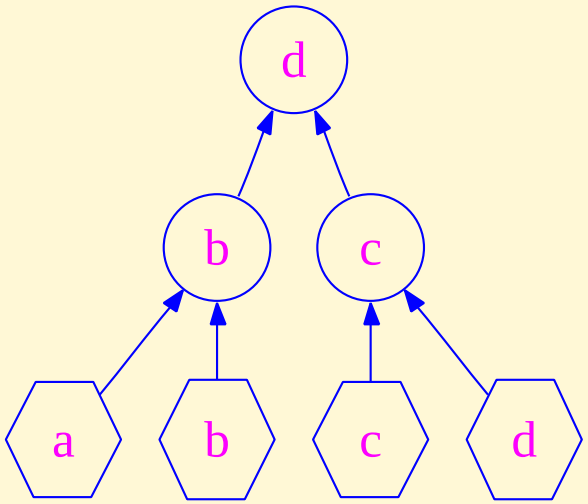
Example: DisHS

- a, c, d send $\langle x = 1 \rangle$, $\langle x = 2 \rangle$, $\langle x = 3 \rangle$ to leaders b and c
- b sends $\langle x = 1 \rangle, \ldots, \langle x = 4 \rangle$ to itself (actually, primitive agent to a leader)
- b, c prune inconsistent pairs and send results to d



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Example: DisHS

b sends consistent pairs to *d*:

a	*b*
1	2
1	3
1	4
2	1
2	3
2	4
3	1
3	2
3	4

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Example: DisHS

c sends consistent pairs to d:

c	d
1	2
1	3
2	1
2	3
3	1
3	2

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Example: DisHS

d prunes inconsistencies using Check queries to *a*, *b*, *c* and produces solutions

a	*b*	*c*	*d*
1	2	3	1
1	2	3	2
1	3	2	1
1	3	2	3
1	4	2	1
1	4	2	3

...

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Example: DisHS

d prunes inconsistencies using Check queries to *a*, *b*, *c* and produces solutions

a	*b*	*c*	*d*
1	2	3	1
1	2	3	2
1	3	2	1
1	3	2	3
1	4	2	1
1	4	2	3

...

Not necessary in this order!

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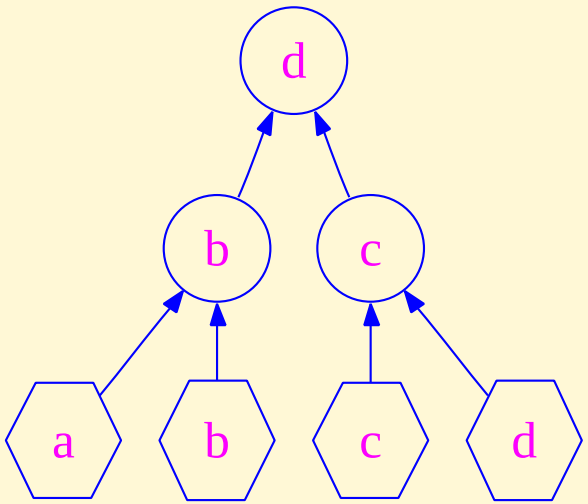
Example: DesRS

- Each agent initiates a backtracking process
- Processes are independent, so let's consider one originating at *a*
- Note: numbers of processes and agents can be independent



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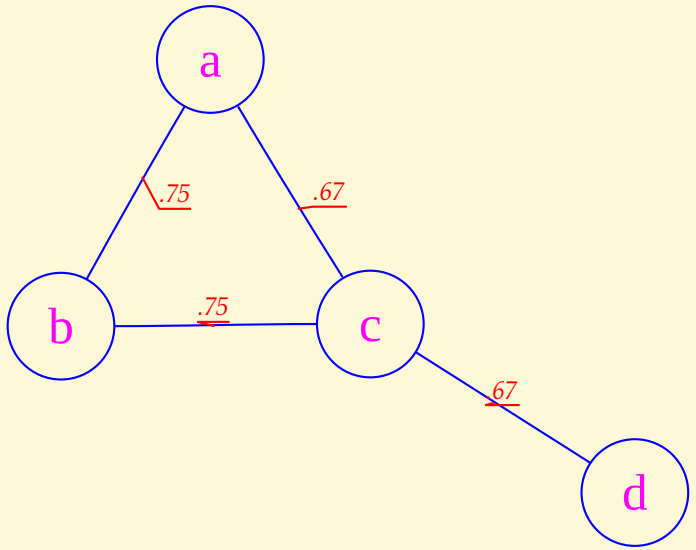
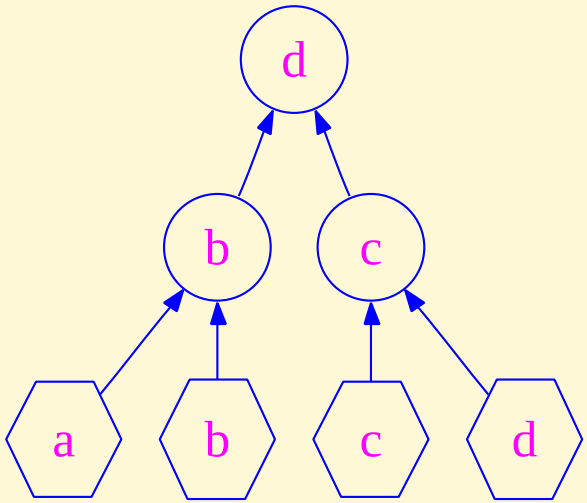
Example: DesRS

- a to leader b: $\langle a = 1 \rangle$
- Leader b forwards to leaf b, which sends $\langle a = 1, b = 2 \rangle$ up
- Leader b forwards tp leaf d via leaders d, c
- d to leader c: $\langle a = 1, b = 2, d = 1 \rangle$





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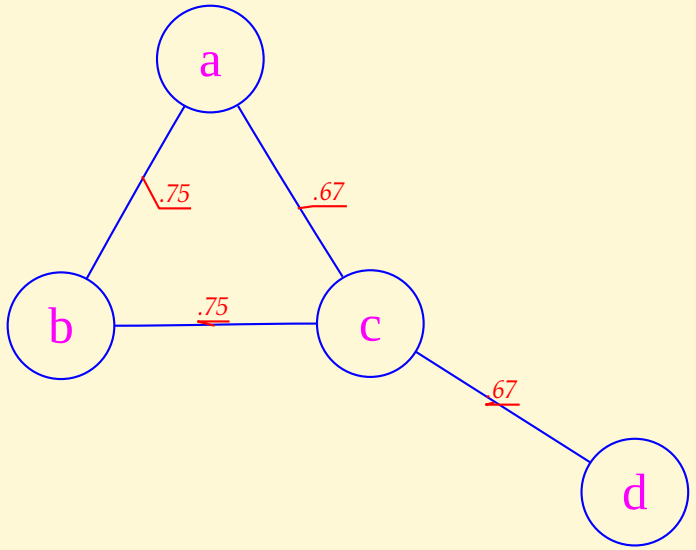
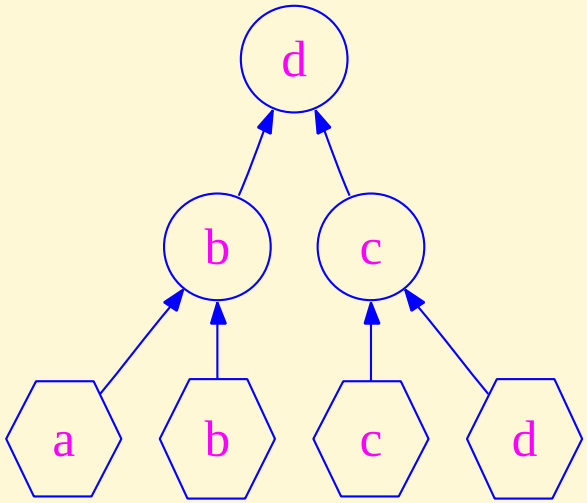
Example: DesRS

- Leaf c sends to leader c a complete solution $\langle a = 1, b = 2, d = 1, c = 3 \rangle$
- Leader c forwards it to USER via leader d





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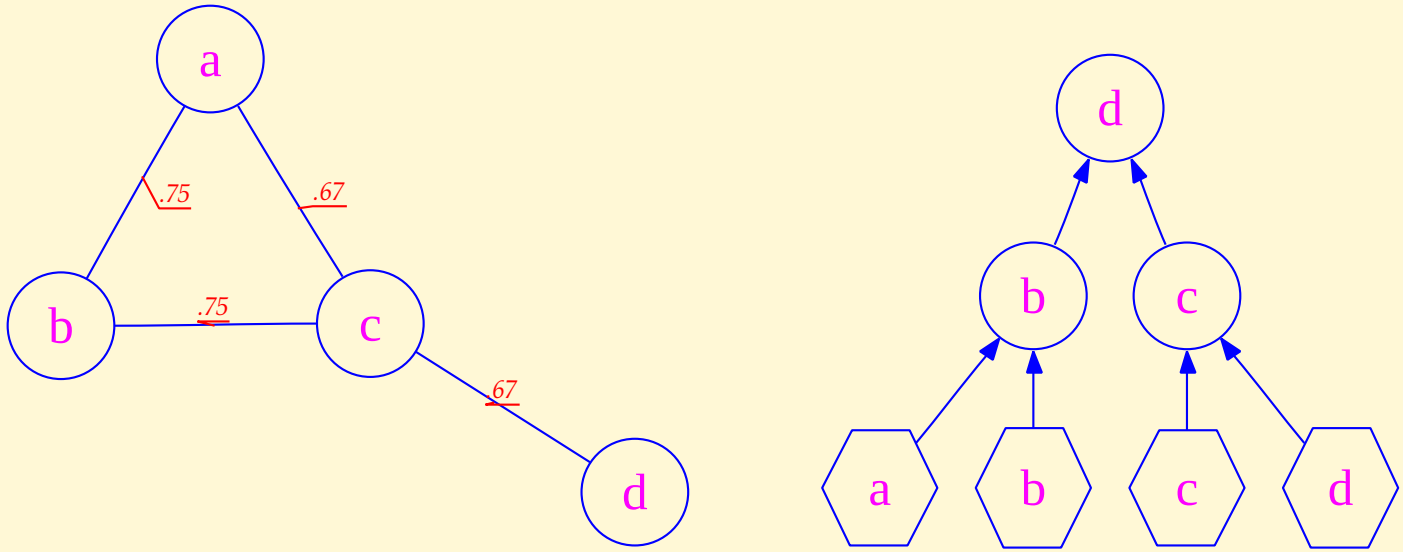
Example: DesRS

- Leaf c sends to leader c a complete solution $\langle a = 1, b = 2, d = 1, c = 3 \rangle$
- Leader c forwards it to USER via leader d



No backtracking! But backtracking works as expected.

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Group Partition

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Algorithm: Partition

- Agents try to join neighbors with minimal weight: Join and NoJoin messages

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Algorithm: Partition

- Agents try to join neighbors with minimal weight: Join and NoJoin messages
- Joining agents exchange component information with Components messages

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Algorithm: Partition

- Agents try to join neighbors with minimal weight: Join and NoJoin messages
- Joining agents exchange component information with Components messages
- Removing joined neighbors and environment for next level: Done messages

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Algorithm: Partition

- Agents try to join neighbors with minimal weight: Join and NoJoin messages
- Joining agents exchange component information with Components messages
- Removing joined neighbors and environment for next level: Done messages
- Leader activation: Leader messages

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Algorithm: Partition

Input : agent s, its neighbors N
Output : components C, pairs, leader, parent ← USER
Locals : Ñ, N_{ldr}, g, level ← ƥ, start ← TRUE, next-level

SEND(s, Leader⟨N, {(s, N, 0)}, TRUE, 0⟩)
loop *forever* **do**
  switch RECEIVE() **do**
    ...

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Algorithm: Partition

case Join⟨t⟩
  if t = g **then**
    SEND(g, Components⟨C⟩)
  else
    SEND(t, NoJoin)
case NoJoin
  g ← SELECT(Ñ)
  SEND(g, Join⟨s⟩)

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Algorithm: Partition

case Components$\langle C_g \rangle$
  if $s \neq g$ **then**
    $\{leader, C\} \leftarrow$ SELECT-LEADER(C, C_g, next-level)
    if parent $=$ USER **then**
      parent $\leftarrow$ leader
  else
    leader $\leftarrow$ s
  foreach $t \in N \cup \{s\}$ **do**
    SEND(t, Done$\langle s, leader \rangle$)

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Algorithm: Partition

case Done$\langle t, \text{leader}_t \rangle$
  if $t \neq s$ **then**
    $N_{ldr} \leftarrow$ UPDATE$(N_{ldr}, N, t, \text{leader}_t)$
  $\tilde{N} \leftarrow \tilde{N} \setminus \{t\}$
  if $\tilde{N} =$ NIL **then**
    level $\leftarrow \mathfrak{p}$
    SEND(leader, Leader$\langle N_{ldr} \setminus \{\text{leader}\}$, C, $s = g$, next-level$\rangle$)

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Algorithm: Partition

case Leader$\langle$N', C', single, level'$\rangle$

  if start **then**

    $N \leftarrow N',\ C \leftarrow C',\ N_{ldr} \leftarrow \text{NIL}$

  else

    $pairs \leftarrow \left\{(\{t, r\}, w) :\ (t, \hat{N}) \in C \ \wedge\ r \in C' \ \wedge\ (r, w) \in \hat{N}\right\}$

    $N \leftarrow \text{COMBINE}(N, N'),\ C \leftarrow C \cup C'$

  $start \leftarrow \neg start \vee single$

  ...

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Algorithm: Partition

```
case Leader⟨N', C', single, level'⟩
    ...
    if start then
        if N = NIL then
            leader ← USER
            foreach t ∈ C do
                SEND(t, Search)
        else
            level ← level',  next-level ← level + 1
            Ñ ← N ∪ {(s, 1.5)}
            g ← SELECT(Ñ)
            SEND(g, Join⟨s⟩)
```


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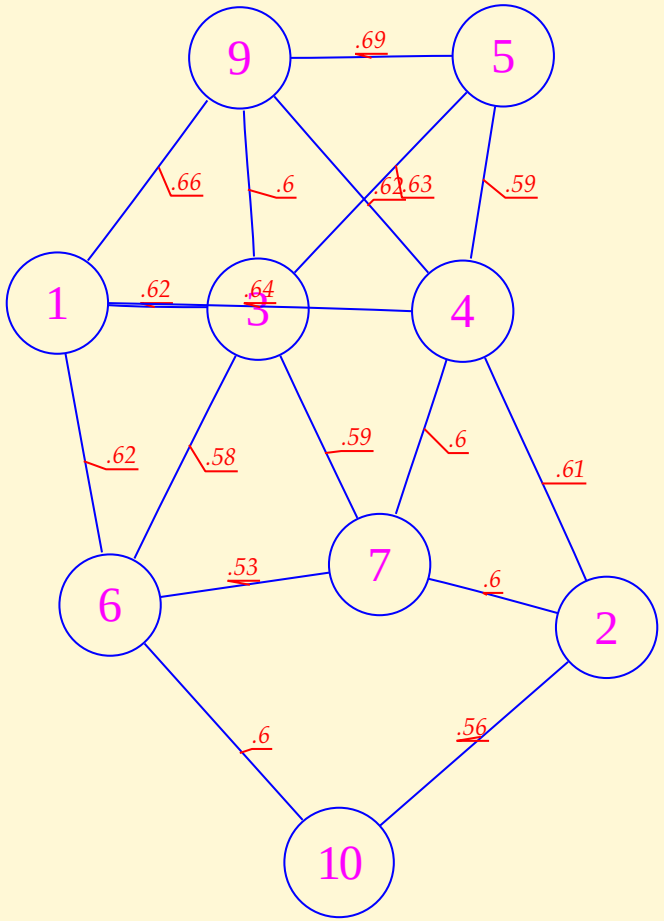
Algorithm: Partition (Example)

- Initial CSP, $p_1 = p_2 = 0.4$.



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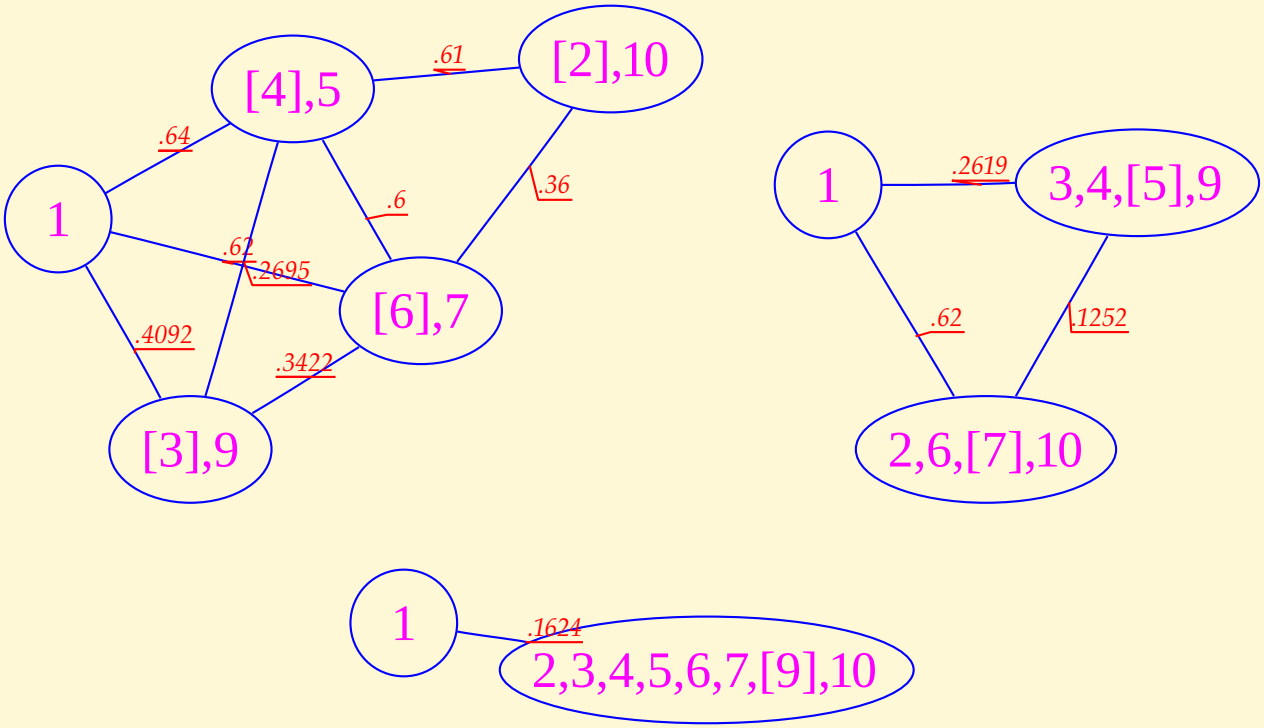
Algorithm: Partition (Example)

- Subsequent levels during partitioning



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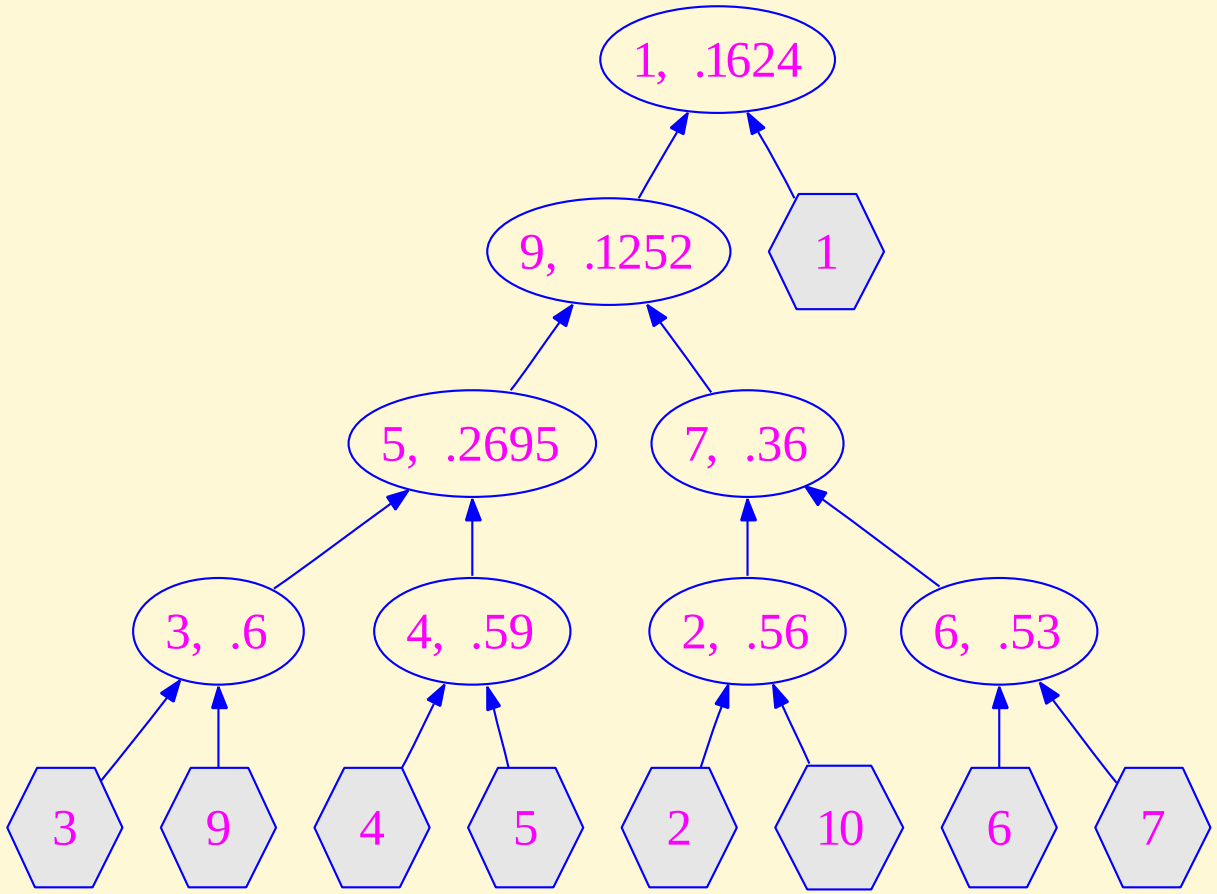
Algorithm: Partition (Example)

- Resulting hierarchy



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Distributed Hierarchical Search

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Algorithm: DisHS

- Agents send consistent PAs up in the hierarchy with Assignment messages

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Algorithm: DisHS

- Agents send consistent PAs up in the hierarchy with Assignment messages
- Representative agents combine PAs

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Algorithm: DisHS

- Agents send consistent PAs up in the hierarchy with Assignment messages
- Representative agents combine PAs
- Check and Answer messages are used for value compatibility queries

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Algorithm: DisHS

- Agents send consistent PAs up in the hierarchy with Assignment messages
- Representative agents combine PAs
- Check and Answer messages are used for value compatibility queries
- **Problem:** message queues saturation

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Algorithm: DisHS

- Agents send consistent PAs up in the hierarchy with Assignment messages
- Representative agents combine PAs
- Check and Answer messages are used for value compatibility queries
- **Problem:** message queues saturation
- **Extensions:** on-demand assignments, query caching, message priority

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Algorithm: DisHS

Input : agent s, partition output, domain D
Output : a global solution is sent to USER
Locals : row $\leftarrow 0$, Pending$[\cdot] \leftarrow 0$, Solutions$[\cdot]$,
Iterator $\leftarrow$ NIL, requests $\leftarrow 1$

loop *forever* **do**
  switch RECEIVE() **do**
    case Search
      level $\leftarrow \mathfrak{s}$
      forall $v \in D$ **do**
        SEND(parent, Assignment$\langle$s, $\{\langle$s, $v\rangle\}\rangle$)
      SEND(parent, Assignment$\langle$s, STOP$\rangle$)
    ...

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Algorithm: DisHS

case Check$\langle$t, row', $\{\langle s, v\rangle, \langle r, w\rangle\}\rangle$
    SEND(t, Answer$\langle$row', CHECK(v, r, w)$\rangle$)
case Answer$\langle$row', ok$\rangle$
    if Pending[row'] $\neq 0$ **then**
        if $\neg$ok **then**
            Pending[row'] $\leftarrow 0$
            requests $\leftarrow$ requests $+ 1$
            PROCESS-REQUEST()
        else
            Pending[row'] $\leftarrow$ Pending[row'] $- 1$
            if Pending[row'] $= 0$ **then**
                SEND(leader, Assignment$\langle$s, Solutions[row']$\rangle$)

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Algorithm: DisHS

case Request
    requests $\leftarrow$ requests $+ 1$
    PROCESS-REQUEST()
case Assignment$\langle$t, partial$\rangle$
    ITERATOR-ADD(Iterator, t, partial)
    PROCESS-REQUEST()

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Descending Requirements Search

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Algorithm: DesRS

- Each agent initiates a backtracking search

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Algorithm: DesRS

- Each agent initiates a backtracking search
- Representative agents serve as routers

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Algorithm: DesRS

- Each agent initiates a backtracking search
- Representative agents serve as routers
- Search processes are independent

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Algorithm: DesRS

- Each agent initiates a backtracking search
- Representative agents serve as routers
- Search processes are independent
- Assignment and Nogood messages

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Algorithm: DesRS

Input : agent s, partition output, domain D, child agents $c_{0,1}$, primitive child indicators $prim_{0,1}$
Output : a global solution is sent to USER
Locals : Id-Map$[\cdot]$

loop *forever* **do**
  switch RECEIVE() **do**
    case Search
      level $\leftarrow \mathfrak{s}$
      SEND(s, Assignment$\langle$s, s, NIL, TRUE$\rangle$)
    ...

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Algorithm: DesRS

case Assignment$\langle$t, id, PA, primitive$\rangle$
  if primitive **then**
    Id-Map[id] $\leftarrow \langle$PA, $D, \emptyset\rangle$
  else if $\exists i$: t $=$ c$_i$ **then**
    if c$_{1-i} \in$ PA **then**
      SEND(leader, Assignment$\langle$s, id, PA, FALSE$\rangle$)
    else
      SEND(c$_{1-i}$, Assignment$\langle$s, id, PA, prim$_{1-i}\rangle$)
  else
    $i \leftarrow$ RANDOM($\{0,1\}$)
    SEND(c$_i$, Assignment$\langle$s, id, PA, prim$_i\rangle$)
case Nogood$\langle$id, exp$\rangle$
  $\langle \cdot, \cdot,$ united-exp$\rangle \leftarrow$ Id-Map[id]
  united-exp $\leftarrow$ united-exp $\cup$ exp

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Algorithm: DesRS

case Assignment$\langle \cdot, \mathsf{id}, \cdot, \textsc{true} \rangle \vee$ Nogood$\langle \mathsf{id}, \cdot \rangle$
  $\langle \mathsf{PA}, \mathsf{Values}, \mathsf{exp} \rangle \leftarrow$ Id-Map$[\mathsf{id}]$
  $v \leftarrow$ NIL
  while $v =$ NIL $\wedge$ Values $\neq \emptyset$ **do**
    $v \leftarrow$ RANDOM(Values)
    Values $\leftarrow$ Values $\setminus \{v\}$
    for $(\mathsf{r} = w) \in \mathsf{PA}$ *(left-to-right, neighbors of* s *only)* **do**
      if CHECK(v, r, w) **then**
        exp $\leftarrow$ exp $\cup \{\mathsf{r}\}$
        $v \leftarrow$ NIL
        break
  ...

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Algorithm: DesRS

case Assignment$\langle \cdot, \mathsf{id}, \cdot, \textsc{true} \rangle \vee$ Nogood$\langle \mathsf{id}, \cdot \rangle$
  ...
  if $v \neq \textsc{nil}$ **then**
    SEND(parent, Assignment$\langle \mathsf{s}, \mathsf{id}, \langle \mathsf{PA}, (\mathsf{s} = v) \rangle, \textsc{false} \rangle$)
  else if exp $\neq \emptyset$ **then**
    for r $\in$ PA *(right-to-left)* **do**
      if r $\in$ exp **then**
        SEND(r, Nogood$\langle \mathsf{id}, \mathsf{exp} \setminus \{\mathsf{r}\} \rangle$)
        break
  else
    SEND(USER, Nogood$\langle \mathsf{id}, \mathsf{exp} \rangle$)

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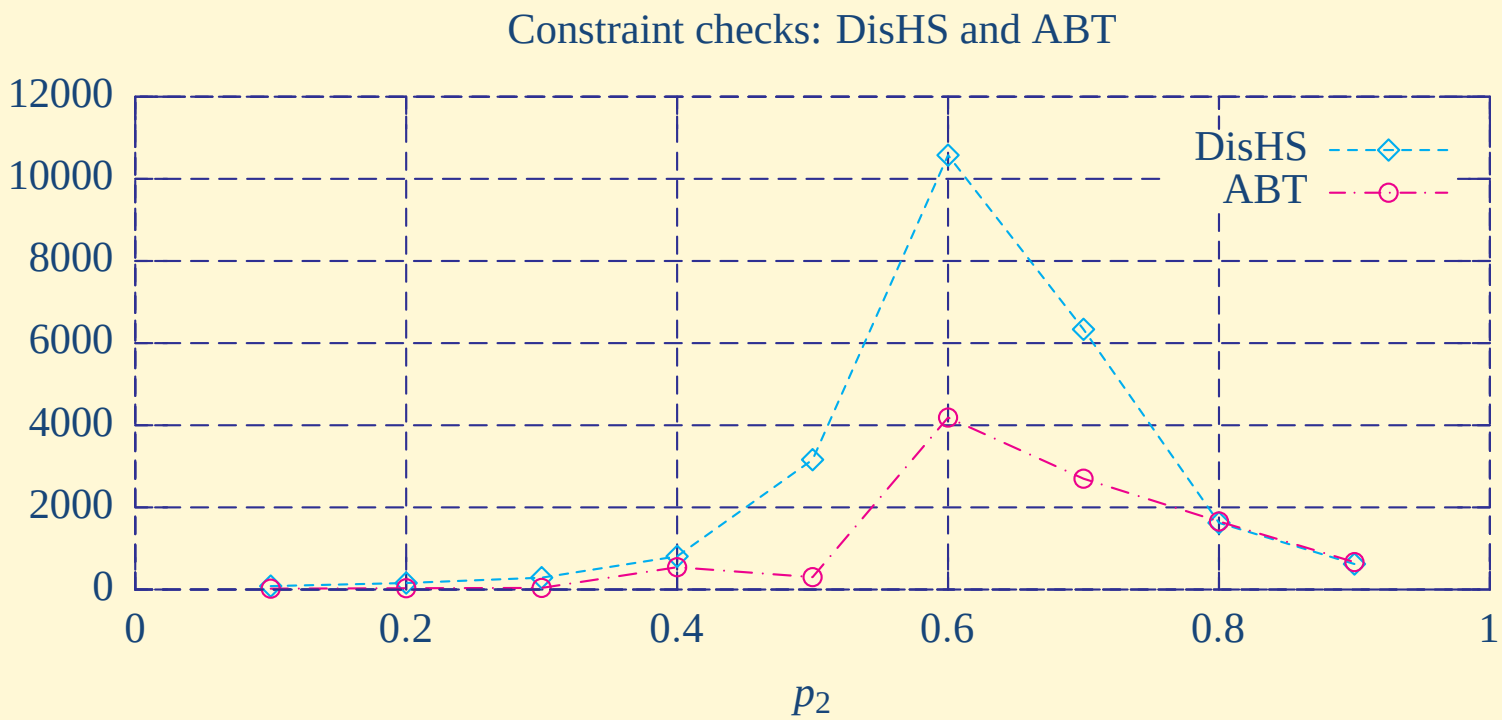
Experimental Results: DisHS

- DisHS, ABT on random problems with 10 agents, domain size of 10, and $p_1 = 0.5$



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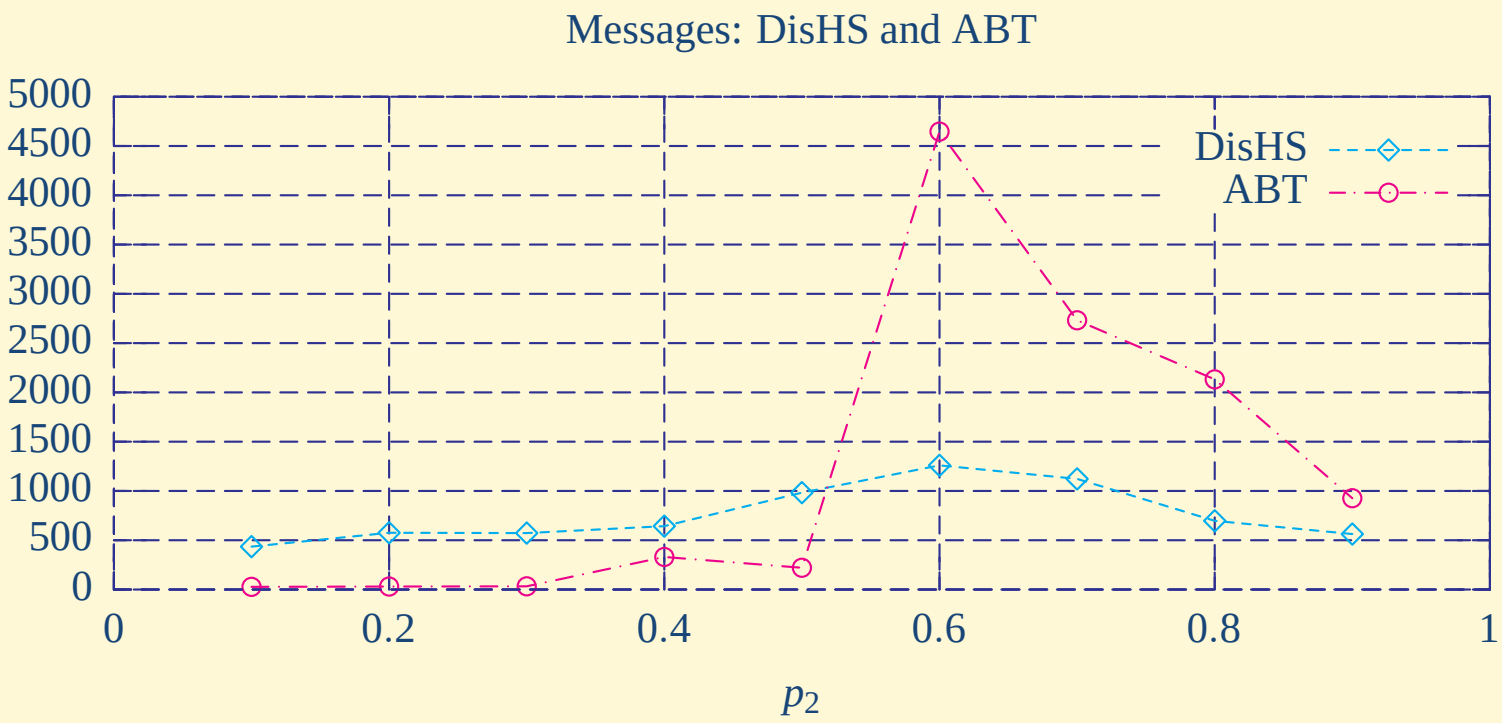

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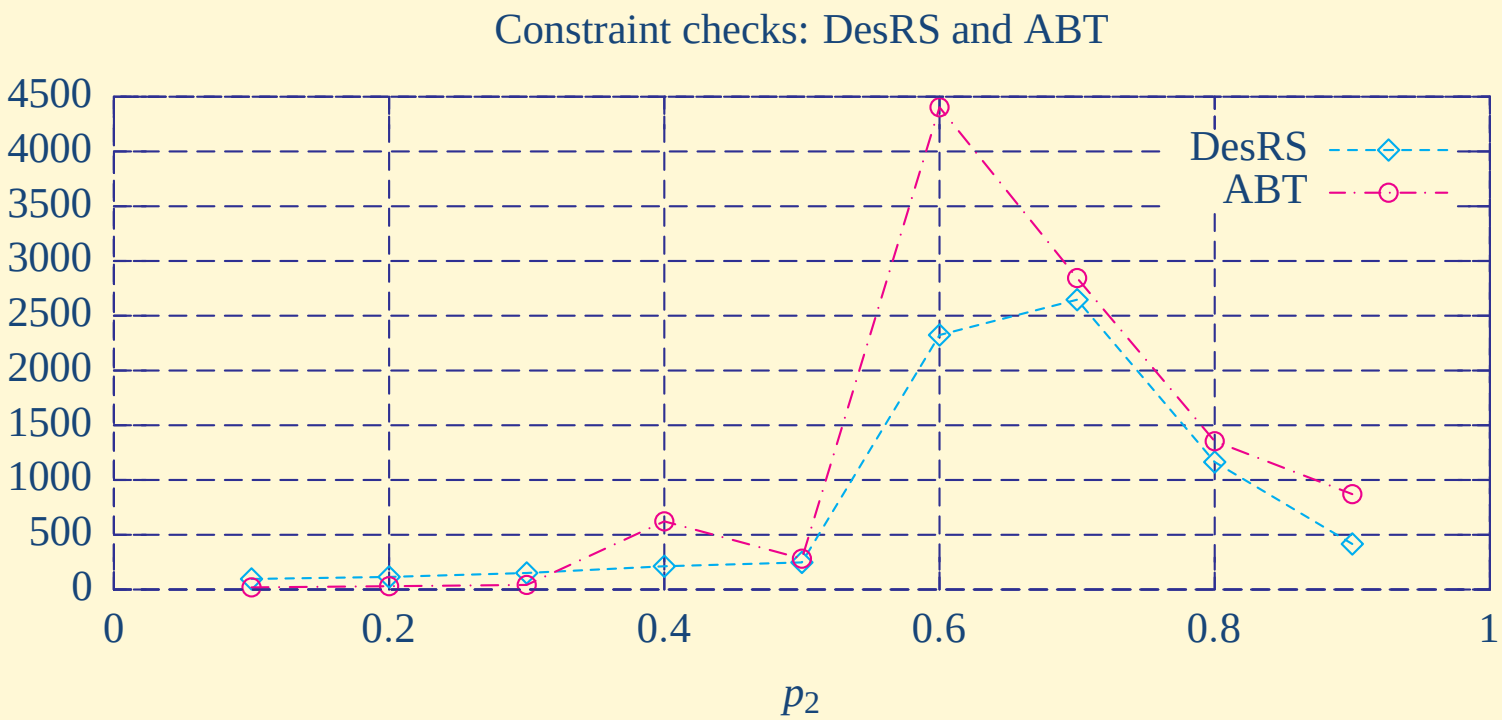
Experimental Results: DesRS

- DesRS, ABT on random problems with 10 agents, domain size of 10, and $p_1 = 0.5$



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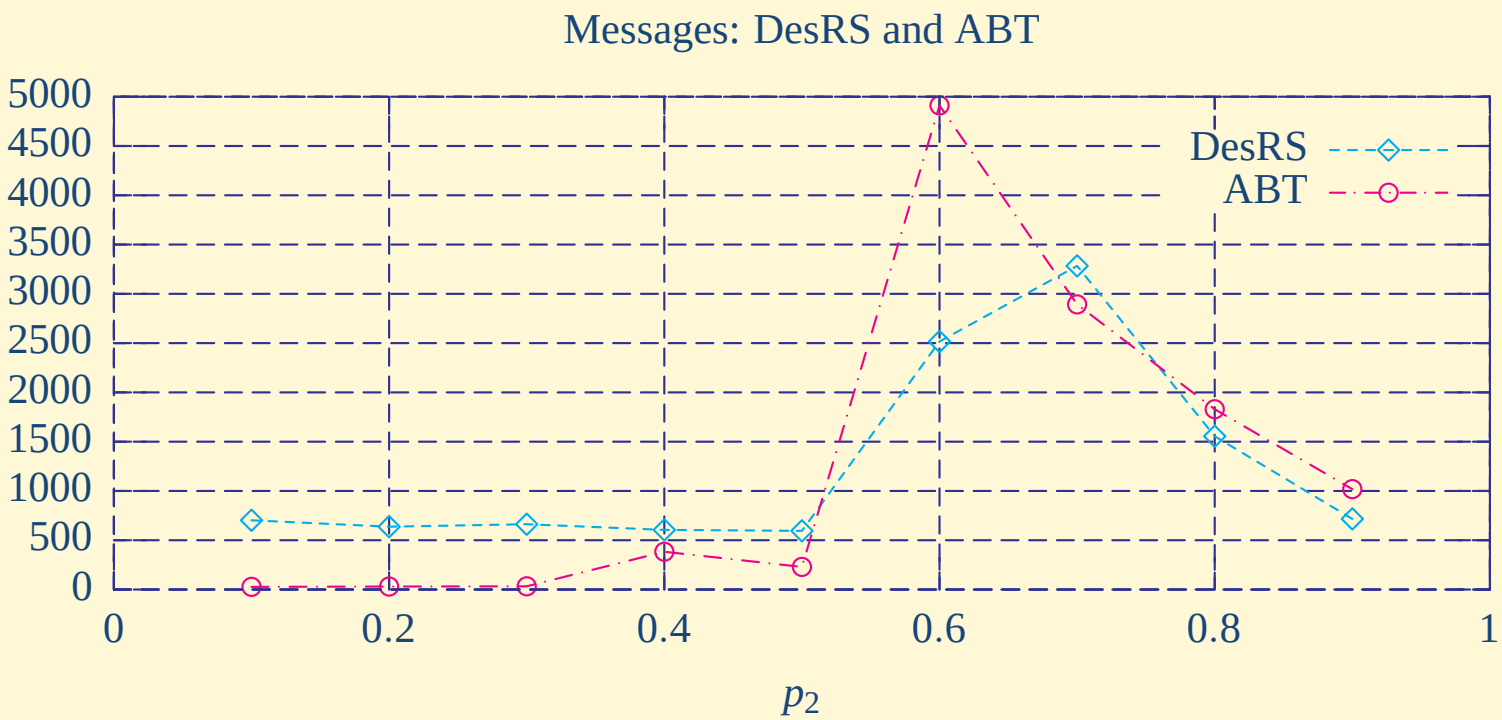
Experimental Results: DesRS

- DesRS, ABT on random problems with 10 agents, domain size of 10, and $p_1 = 0.5$



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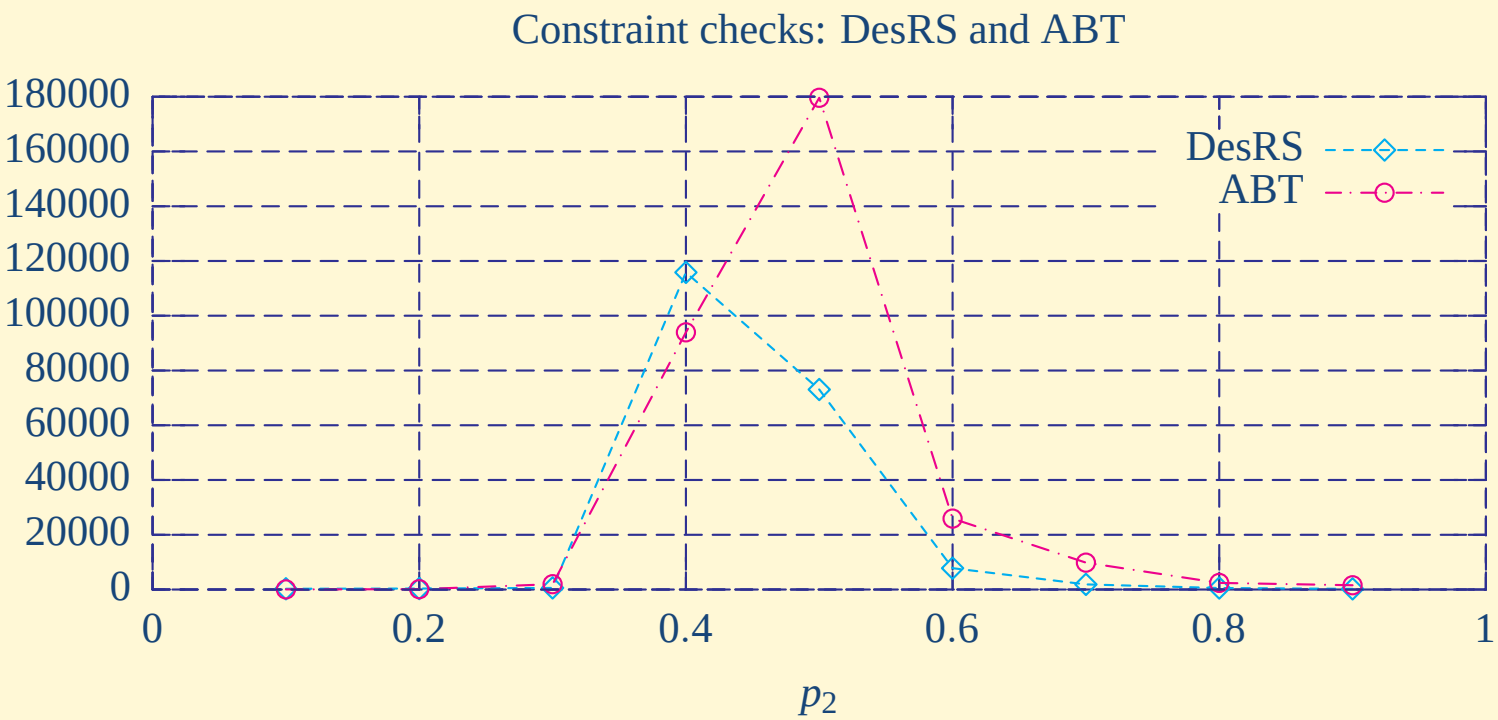
Experimental Results: DesRS

- DesRS, ABT on random problems with 20 agents, domain size of 10, and $p_1 = 0.4$



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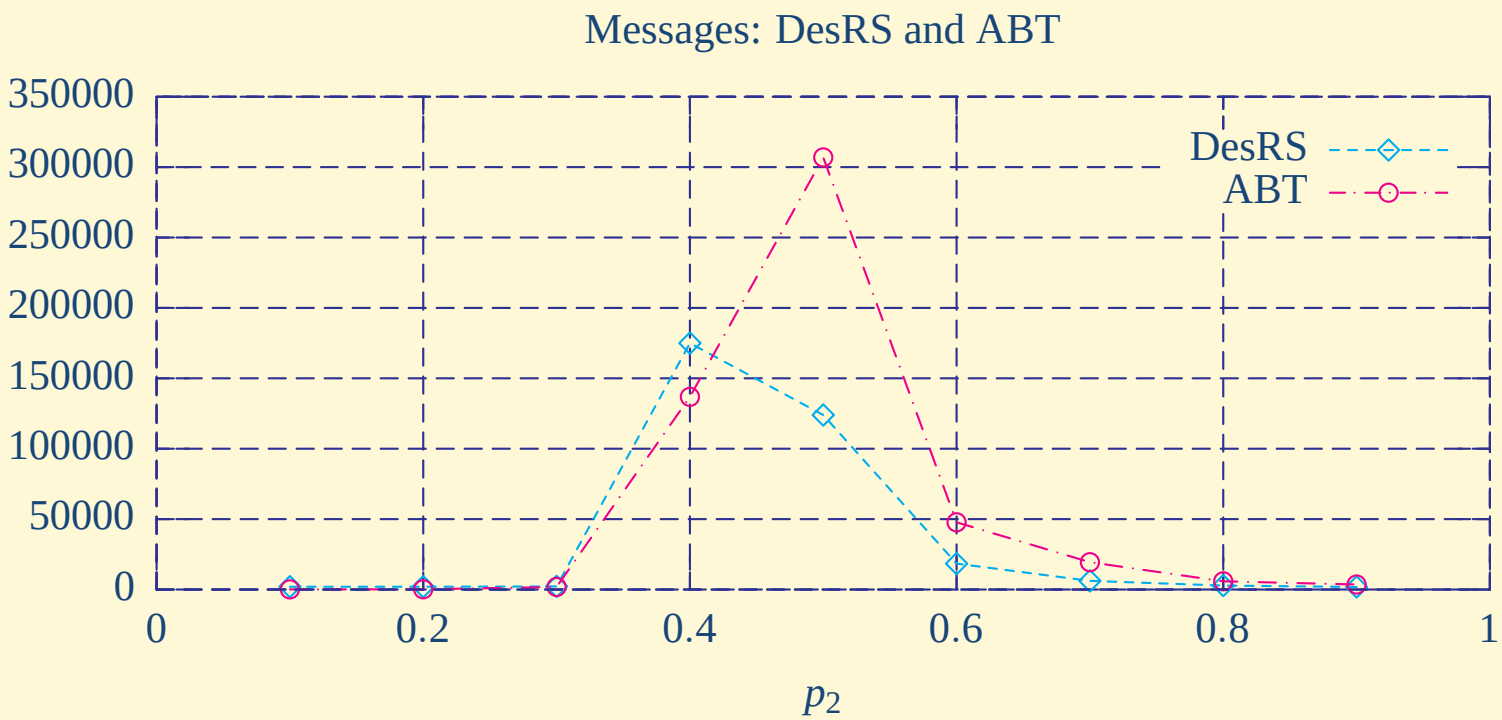
Experimental Results: DesRS

- DesRS, ABT on random problems with 20 agents, domain size of 10, and $p_1 = 0.4$



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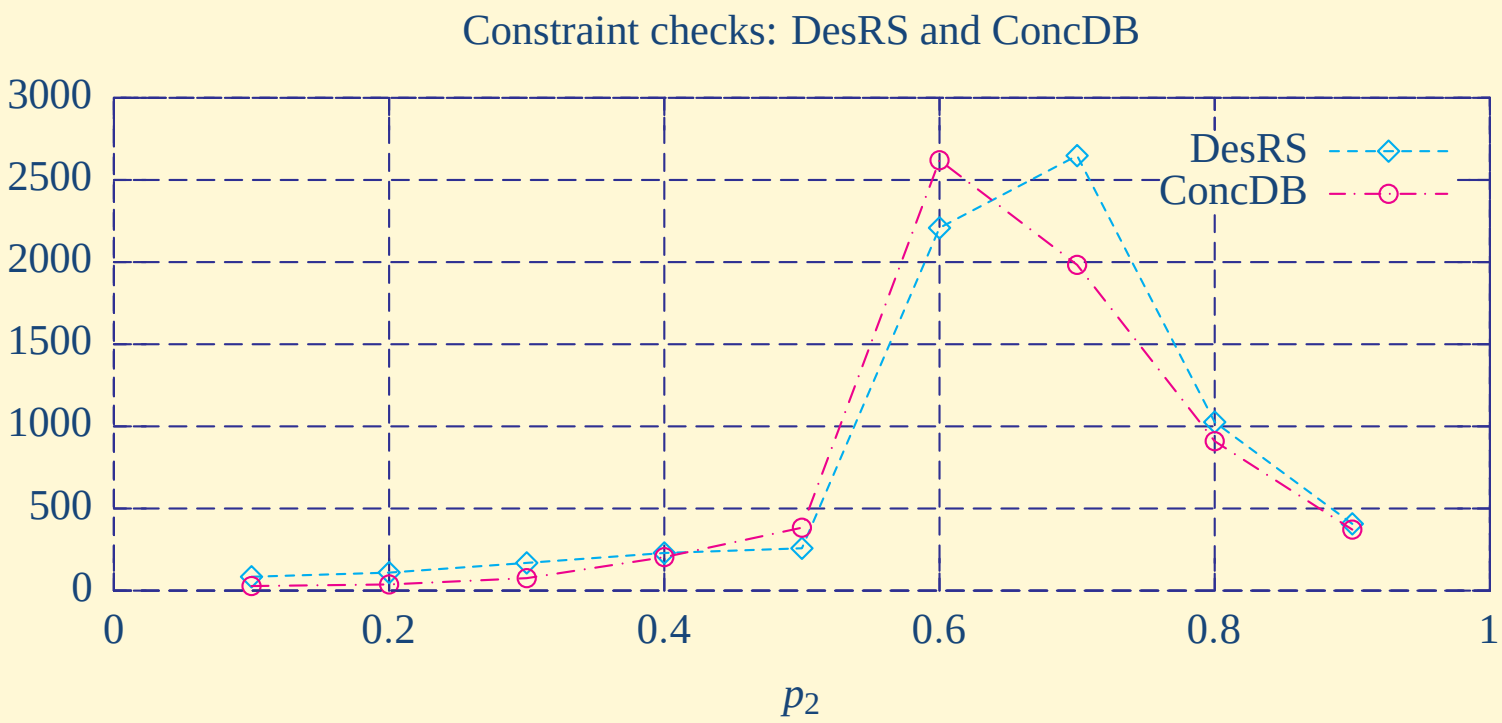
Experimental Results: DesRS

- DesRS, ConcDB [Zivan and Meisels, 2006] on random problems with 10 agents, domain size of 10, and $p_1 = 0.5$



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Experimental Results: DesRS

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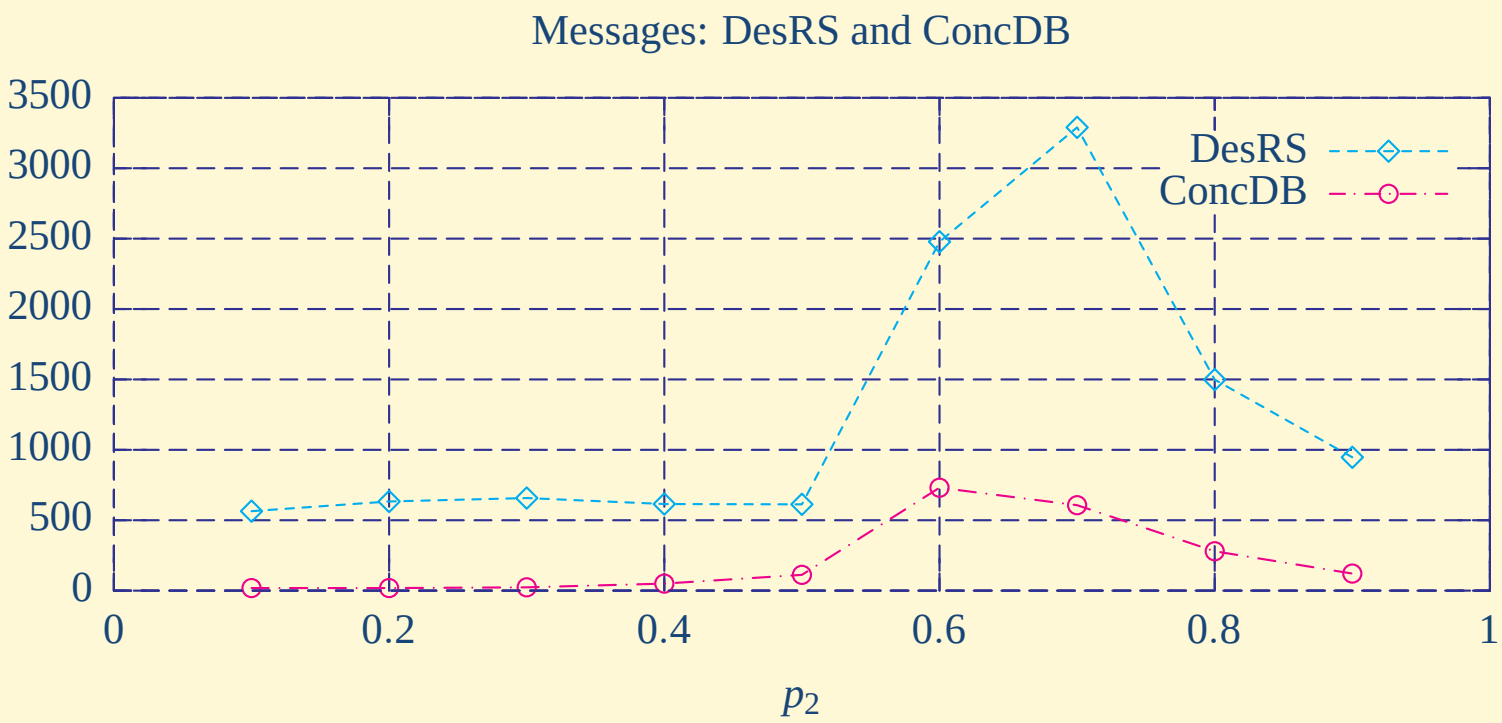

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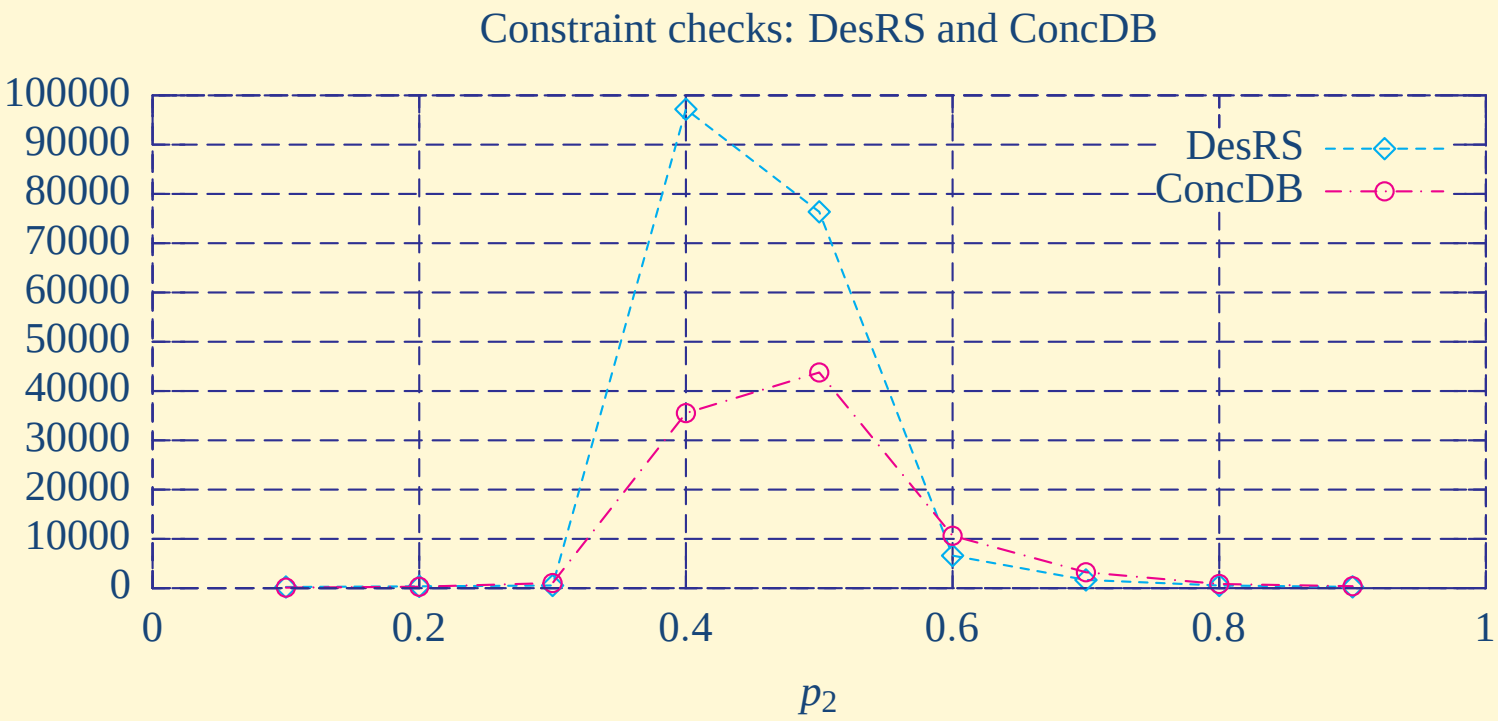
Experimental Results: DesRS

- DesRS, ConcDB on random problems with 20 agents, domain size of 10, and $p_1 = 0.4$



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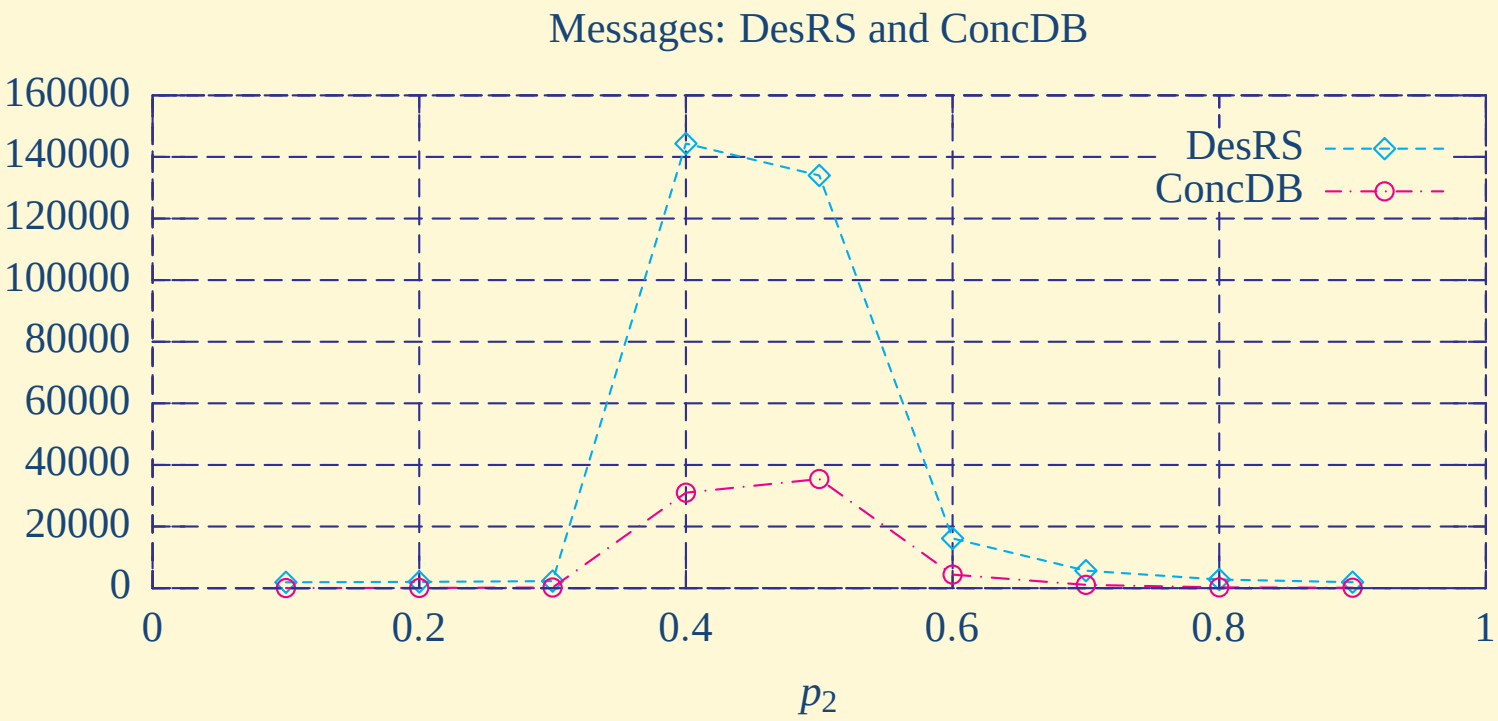
Experimental Results: DesRS

- DesRS, ConcDB on random problems with 20 agents, domain size of 10, and $p_1 = 0.4$



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Importance of partition: DisHS vs. AntiDisHS

- What if we change the order of neighbors during partition (10 agents, 10 values, $p_1 = 0.5$)?

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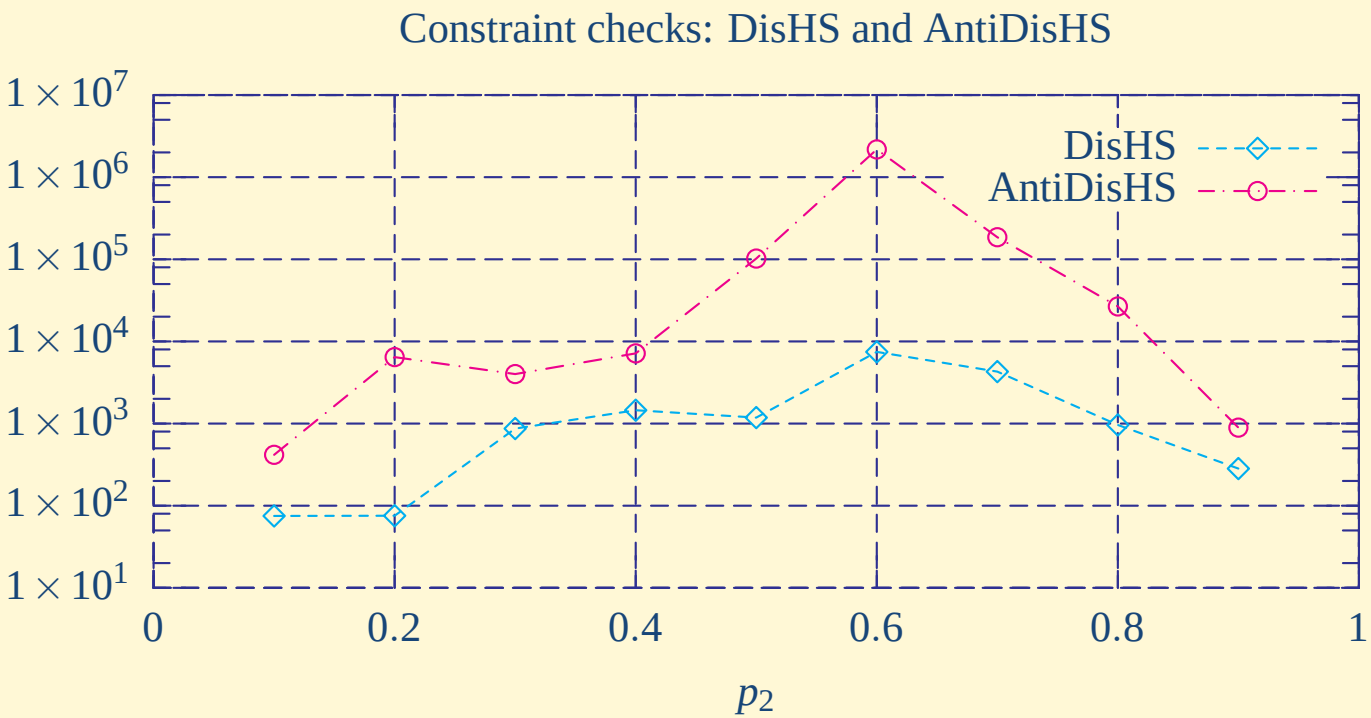

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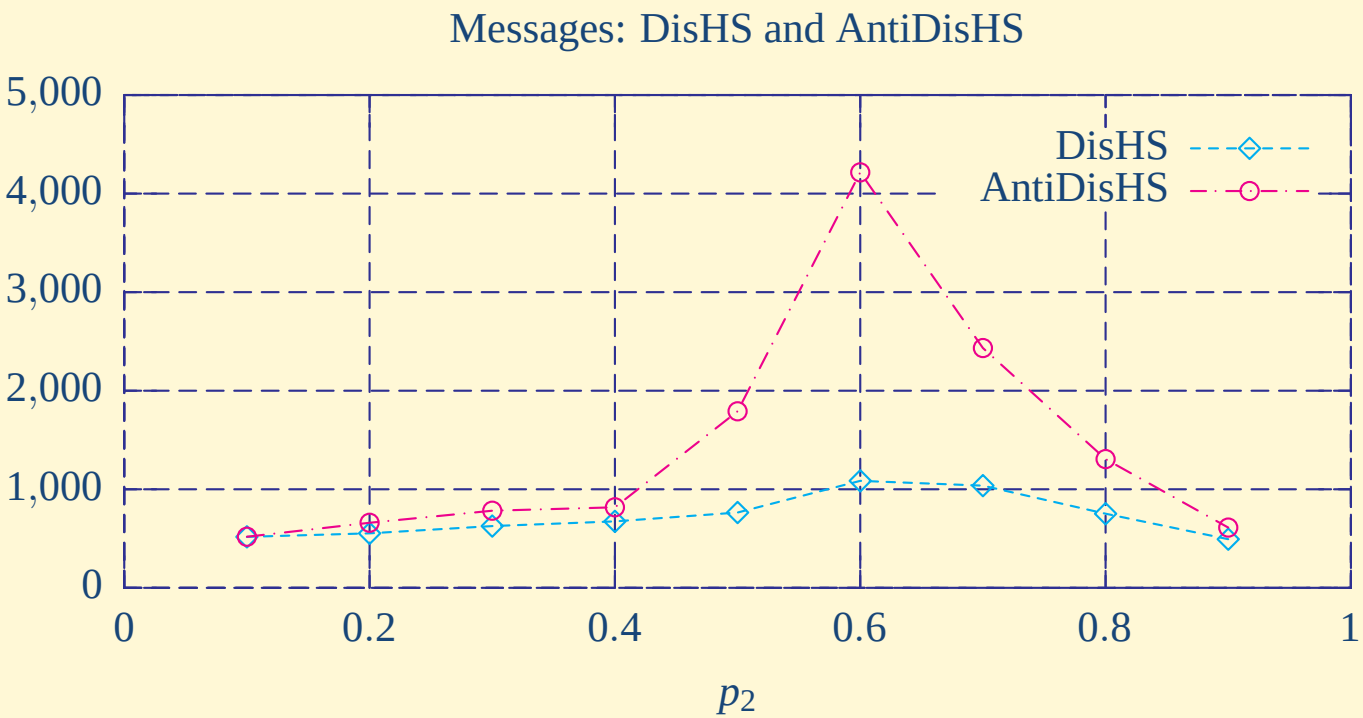

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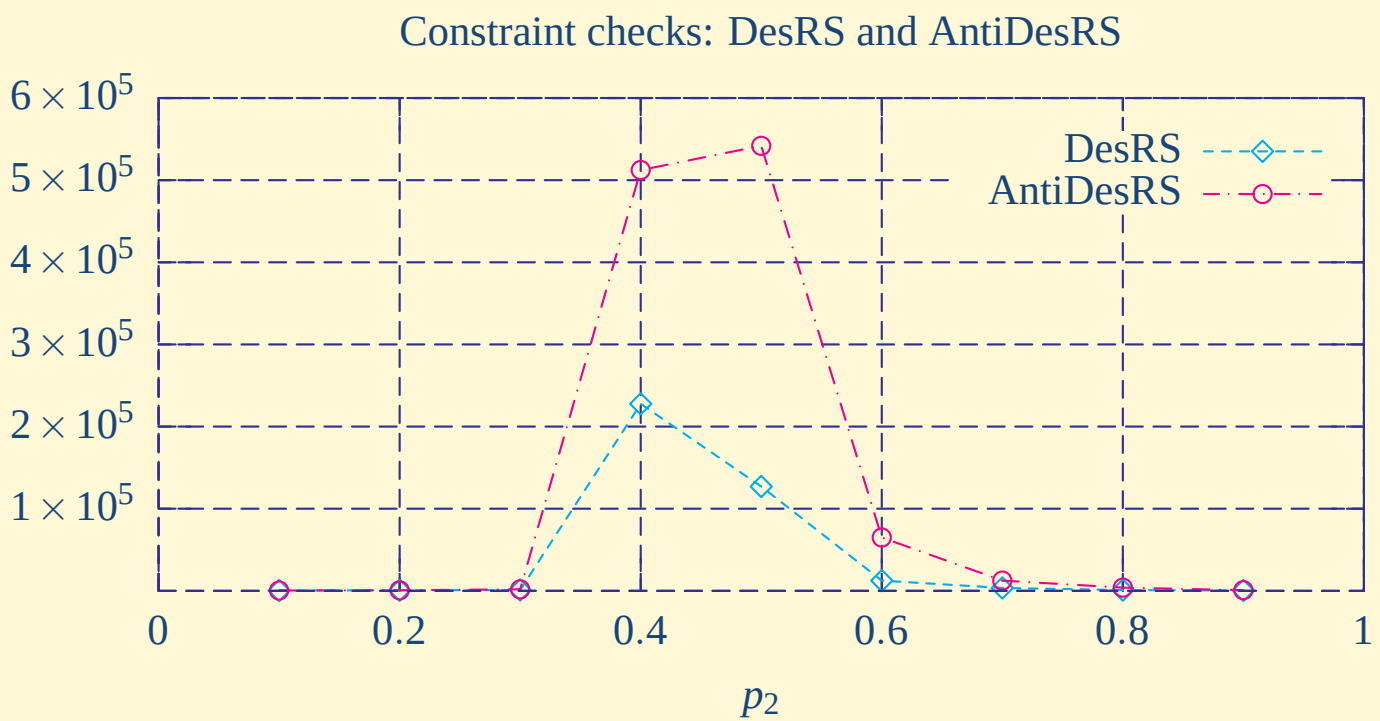

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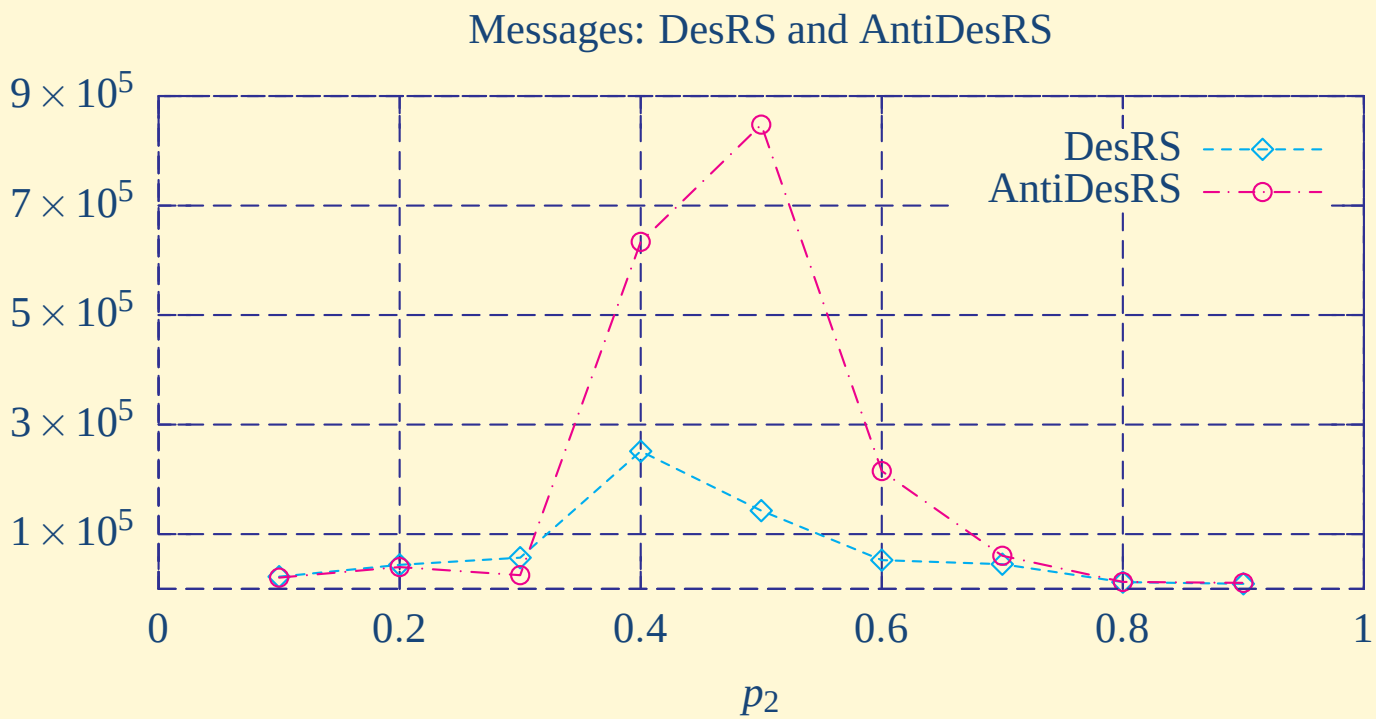

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Partition: Additional applications

- Influence in social networks

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Partition: Additional applications

- Influence in social networks
- Load balancing

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Partition: Additional applications

- Influence in social networks
- Load balancing
- Partition is fast!

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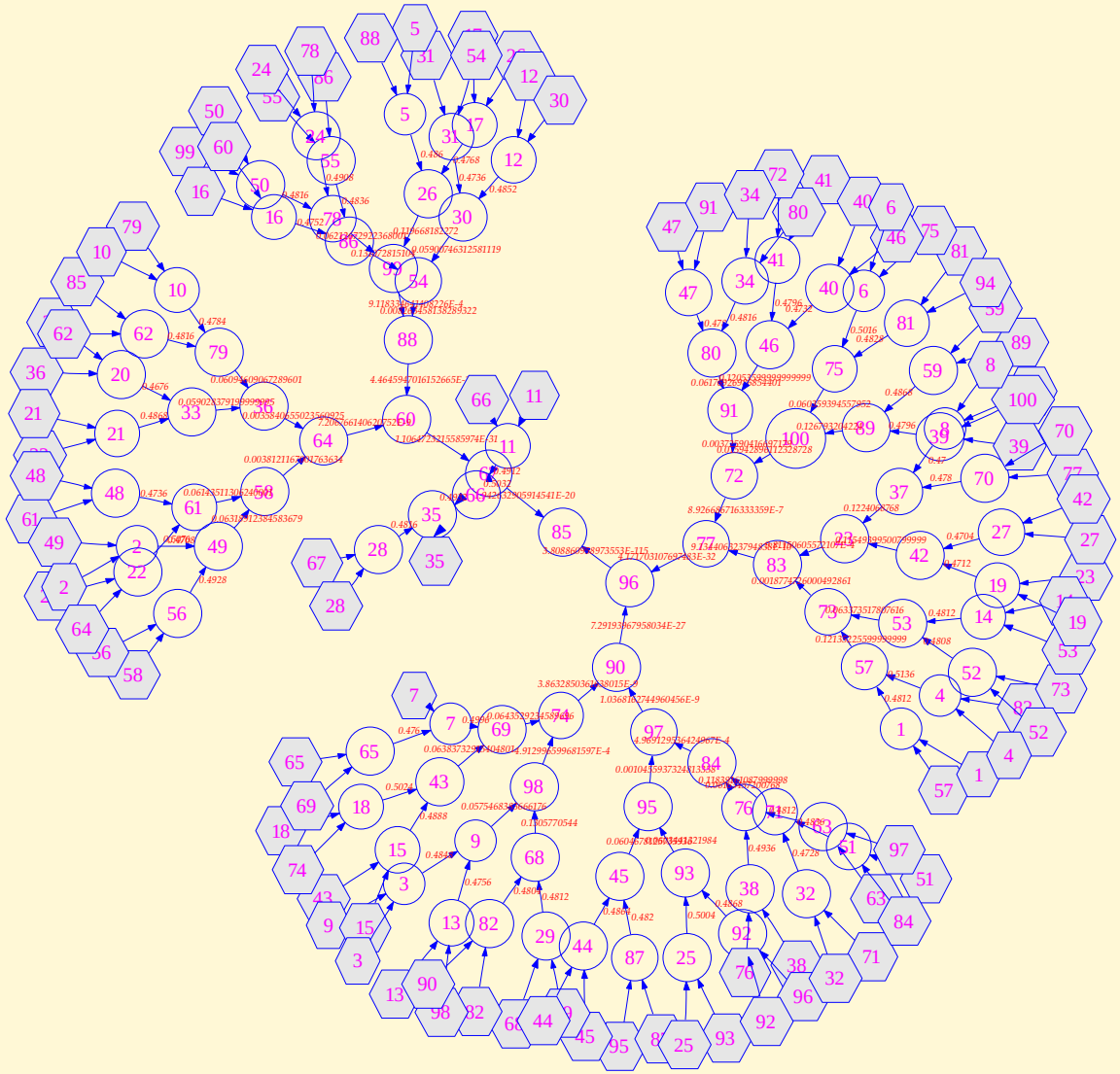
Partition: Additional applications



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